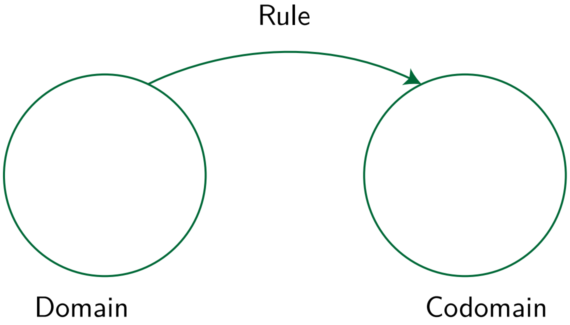
## Goals

Everything in the start of this lesson is technically about *mappings* of which *functions* are only a part. This makes no difference to you for the moment, however, so don't try to understand the difference until it is presented at the end.
We will

- learn terminology for functions
- learn to read function notation
- practice using function notation
- test for function inverses (solve functions)

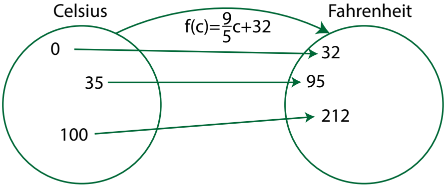
# Terminology: Mappings

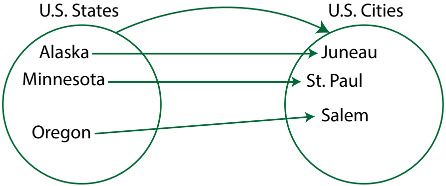
A *mapping* connects items from a collection called the *domain* to items in a collection called the *codomain*. The method of mapping is called the *rule* (the pattern).



# Terminology: Mappings

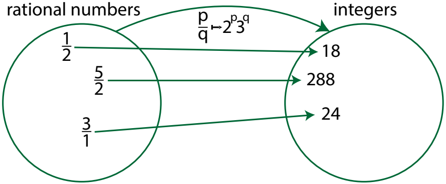
The first two examples illustrate the concept of mapping with applications. Note that not all functions involve numbers.

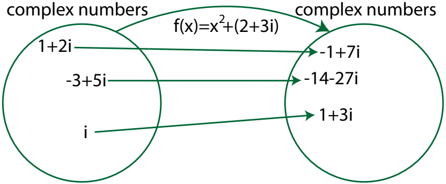


# Terminology: Mappings

The first example illustrates that the domain and codomain do not have to be the same (different types of numbers). The second reminds us that we can use complex numbers as well.



# Example

When you encounter functions in future math and non-math classes, you will see the kind of notation below. You will be expected to figure out the domain and codomain and make sense of the rule on your own.

$$\begin{aligned} f(x) &= 2(x-3)^2+5. \\ g(x) &= \frac{x^2-1}{x}. \\ h(x) &= \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases} \end{aligned}$$

# Example 1

$$f(x) = 2(x-3)^2 + 5.$$

- Domain: This formula involves arithmetic so the domain is (real) numbers. Note: unless explicitly stated we will not use complex numbers in this course. Note there are no restrictions on which numbers that can be plugged into this mapping.
- Codomain: If a number is plugged into this mapping, a number comes out. The codomain is also (real) numbers.
- Range: While codomain refers to the type of object output, range refers to the set that actually come out. For this function they are $[5, \infty)$ because the square term is never negative. Calculus will provide techniques for finding the range: it is not a task for this course.

# Example 2

$$g(x) = \frac{x^2 - 1}{x}.$$

- Domain: This formula involves arithmetic so the domain is (real) numbers. However, if $x = 0$ division by zero occurs, so the domain is all real numbers except 0.
- Codomain: This mapping also outputs (real) numbers.

# Example 3

$$h(x) = \begin{cases} x & \text{if} \quad x \geq 0 \\ -x & \text{if} \quad x < 0 \end{cases}$$

This notation indicates that more than one rule is needed to express the mapping. This is called *piecewise defined* mapping. The notation is used as following. To evaluate $h(-7)$ note that $-7 < 0$ so $h(-7) = -(-7) = 7$ from the second rule. However for $h(5)$ note that $5 \geq 0$ so $h(5) = 5$ using the first rule.

- Domain: Only numbers can be negated, so the domain is (real) numbers.
- Codomain: This returns the number or its negative, so the codomain is (real) numbers.

# Example 4

$$j(x) = \frac{1+i}{\sqrt{2}}x + (1+2i).$$

- Domain: The rule involves complex numbers, so the domain is complex numbers.
- Codomain: Arithmetic with complex numbers will produce complex numbers.

Try the first set of practice problems.

# Example

Function notation can be used as follows.

$$
\begin{aligned}
f(x) &= 2(x-3)^2+5. \\
f(5) &= 2(5-3)^2+5 \\
&= 13. \\
f(a-2) &= 2([a-2]-3)^2+5 \\
&= 2(a-5)^2+5.
\end{aligned}
$$

# Example

$$
\begin{aligned}
f(x) &= 2(x-3)^2+5. \\
g(x) &= 7x-11. \\
g(f(x)) &= 7f(x)-11 \\
&= 7[2(x-3)^2+5]-11 \\
&= 14(x-3)^2+35-11 \\
&= 14(x-3)^2+24. \\
f(x)g(x) &= [2(x-3)^2+5][7x-11] \\
&= 14x(x-3)^2+35x-22(x-3)^2-55.
\end{aligned}
$$

# Example

$$\begin{aligned}
f(x) &= 2(x-3)^2+5. \\
f(7-2i) &= 2([7-2i]-3)^2+5 \\
&= 2(4-2i)^2+5 \\
&= 2[16-16i+4i^2]+5 \\
&= 2[16-16i-4]+5 \\
&= 2[12-16i]+5 \\
&= 29-32i.
\end{aligned}$$

Try the second set of practice problems.

# Definition

Some mappings are *functions*. To understand what type of mappings qualify as functions do the following.

1. Look at the next slide with examples of mappings that are functions.
2. Look at the following slide with examples of mappings that are not functions.
3. On the following (third) slide, determine which of the mappings are functions.
4. Everyone's results will be used in class to illustrate the typical definition.

# Definition

The following mappings are all functions.

| Domain | Codomain | Rule |
|---|---|---|
| real numbers | real numbers | $x$ maps to $3x^2 + 3x + 7$ |
| integers | complex numbers | $n$ maps to $i^n(2+3i)$ |
| U.S. states | U.S. cities | state maps to its capital |
| $(-\infty, 0) \cup (0, \infty)$ | real numbers | $x$ maps to $\frac{x-3}{x}$ |
| Degrees $[0, 360)$ | real numbers | $d$ maps to $d$ |

# Definition

The following mappings are all **not** functions.

| Domain | Codomain | Rule |
|---|---|---|
| real numbers | real numbers | $x$ maps to $y$ if $x = y^2$ |
| real numbers | complex numbers | number $a$ maps to $a + bi$ for all numbers $b$ |
| mothers | people | mother maps to child |
| people | jobs | person maps to job they have held |

# Definition

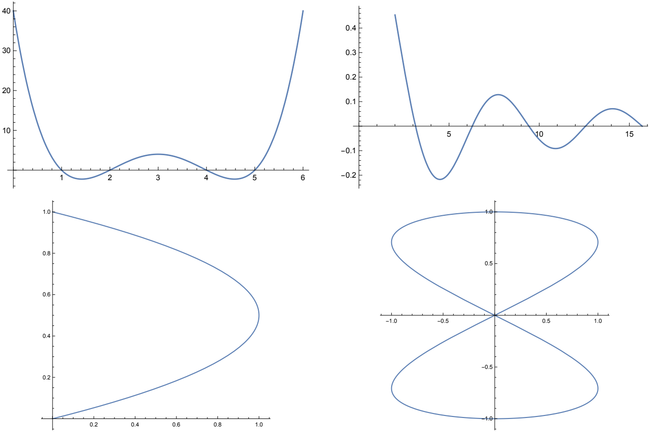
Which of the following mappings are all functions?

| Domain | Codomain | Rule ($x$ maps to) |
|---|---|---|
| real numbers | real numbers | $(x-2)(x-4)(x-7)$ |
| complex numbers | complex numbers | $(2-3i)x+(5-4i)$ |
| real numbers | real numbers | numbers $y$ such that $\lvert y\rvert = x$. |
| $(-\infty, 7)\cup(7,\infty)$ | real numbers | $\frac{(x+3)(x-5)}{x-7}$ |

## Do It Anyway

Often in classes and textbooks shortcuts are used for determining if a graph represents a function. These lead to a false impression that we cannot use functions to study these curves. The top two graphs on the next slide represent functions using traditional choices. The bottom right is a function, but requires a different approach that will be presented in calculus. For the bottom left example determine what choice of domain and codomain would make this a function. This technique will appear regularly in calculus.

# Do It Anyway


40
30
20
10
1
2
3
4
5
6
0.4
0.3
0.2
0.1
−0.1
−0.2
5
10
15
1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
1.0
1.0
0.5
−1.0
−0.5
0.5
1.0
−0.5
−1.0

## Solving Example

In addition to evaluating functions as above, we can also solve to find which inputs produce a particular result.

$$
\begin{aligned}
h(x) &= 11 - \frac{2-x}{5}. \\
h(x) &= 7. \\
7 &= 11 - \frac{2-x}{5}. \\
-4 &= -\frac{2-x}{5}. \\
20 &= 2 - x. \\
18 &= -x. \\
-18 &= x.
\end{aligned}
$$

# Solving Example

$$\begin{aligned}
f(x) &= 2(x-3)^2+5. \\
f(x) &= 6. \\
6 &= 2(x-3)^2+5. \\
1 &= 2(x-3)^2. \\
\frac{1}{2} &= (x-3)^2. \\
\sqrt{\frac{1}{2}} &= |x-3|.
\end{aligned}$$

# Solving Example

$$\sqrt{\tfrac{1}{2}} = |x-3| \qquad .$$

$$\sqrt{\tfrac{1}{2}} = x - 3. \qquad\qquad \sqrt{\tfrac{1}{2}} = -(x-3).$$

$$\sqrt{\tfrac{1}{2}} = -x + 3.$$

$$-3 + \frac{1}{\sqrt{2}} = -x.$$

$$3 + \frac{1}{\sqrt{2}} = x. \qquad\qquad 3 - \frac{1}{\sqrt{2}} = x.$$

## Solving Example

$$
\begin{aligned}
g(x) &= (2+3i)x-(7-2i). \\
g(x) &= i. \\
i &= (2+3i)x-(7-2i). \\
7-i &= (2+3i)x. \\
\frac{7-i}{2+3i} &= x. \\
\frac{7-i}{2+3i}\cdot\frac{2-3i}{2-3i} &= x. \\
\frac{14-21i-2i+3i^2}{4-6i+6i-9i^2} &= x. \\
\frac{11-23i}{13} &= x.
\end{aligned}
$$

# Inverse

In the examples above, when the functions were solved did all of them give a unique (single) result? A function that always gives a unique result when solved is said to have a *function inverse*. It also is called *one-to-one*.

Determine which of the following functions have a function inverse.

- $f(x) = \frac{11-7x}{15}$.
- $g(x) = 3(x+5)^2 - 7$.
- $h(x) = (x-2)(x-[2+i])(x-[2-i])$.
- $j(x) = |7-x| + 2$.

Complete the practice problems.