



Department of Mathematical Sciences

UNIVERSITY of ALASKA ANCHORAGE

Calculating a Derivative by Definition

$$f(x) = \frac{1}{\sqrt{1-x}}.$$

$$\begin{aligned}
 f'(a) &= \\
 \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} &= && \text{definition of derivative} \\
 \lim_{x \rightarrow a} \frac{\frac{1}{\sqrt{1-x}} - \frac{1}{\sqrt{1-a}}}{x - a} &= \\
 \lim_{x \rightarrow a} \frac{\frac{x - a}{\sqrt{1-a}\sqrt{1-x}}}{x - a} &= && \text{common denominator} \\
 \lim_{x \rightarrow a} \frac{\frac{\sqrt{1-a}-\sqrt{1-x}}{\sqrt{1-x}\sqrt{1-a}} \cdot \frac{\sqrt{1-a} + \sqrt{1-x}}{\sqrt{1-a} + \sqrt{1-x}}}{(1-a) - (1-x)} &= && \text{using a conjugate} \\
 \lim_{x \rightarrow a} \frac{\frac{\sqrt{1-a}-\sqrt{1-x}}{x-a} \cdot \frac{\sqrt{1-a} + \sqrt{1-x}}{\sqrt{1-a} + \sqrt{1-x}}}{(1-a) - (1-x)} &= \\
 \lim_{x \rightarrow a} \frac{\frac{\sqrt{1-a}-\sqrt{1-x}}{x-a} \cdot \frac{\sqrt{1-a} + \sqrt{1-x}}{\sqrt{1-a} + \sqrt{1-x}}}{(1-a) - (1-x)} &= \\
 \lim_{x \rightarrow a} \frac{1}{(1-a)\sqrt{1-x} + (1-x)\sqrt{1-a}} &= \frac{1}{2(1-a)^{3/2}}.
 \end{aligned}$$