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Derivative of x^2 versus derivative of 2^x

x^2	2^x
polynomial term	exponential function

Note first why the expression on the left is a polynomial and the expression on the right is an exponential function. x^2 has the variable (that which is changing) in the base. 2^x has the variable (that which is changing) in the exponent. Because derivatives calculate rates of change, and these two expressions have change in different locations, their derivatives can be expected to be different.
For x^2 you learned that the derivative is $2x^{2-1} = 2x$. However, the proof provided fails for x^π. For both polynomial terms and exponential functions we need the definition of exponent provided by $a^b \equiv e^{b \ln a}$.

$f(x)$	$=$	x^2	$g(x)$	$=$	2^x
	$=$	$e^{2 \ln x}$.		$=$	$e^{x \ln 2}$.
$f'(x)$	$=$	$e^{2 \ln x}\left(2\frac{1}{x}\right)$	$g'(x)$	$=$	$e^{x \ln 2}(\ln 2)$
	$=$	$x^2\left(\frac{2}{x}\right)$		$=$	$2^x(\ln 2)$.
	$=$	$2x$.			

Thus we see that the polynomial derivative rule works for any (constant) exponent, and polynomial terms and exponential functions have different derivatives.