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Correct Handling of Known Infinite Limit Forms

$$\lim_{x \to 0^+} \frac{3x^2 + 2}{x}$$

The following approach does not work although all steps are correct.

$$
\begin{aligned}
\lim_{x \to 0^+} \frac{3x^2 + 2}{x} &= \quad \text{division} \\
\frac{\lim_{x \to 0^+} 3x^2 + 2}{\lim_{x \to 0^+} x} &= \quad \text{addition} \\
\frac{\lim_{x \to 0^+} 3x^2 + \lim_{x \to 0^+} 2}{\lim_{x \to 0^+} x} &= \quad \text{multiplication} \\
\frac{\left(\lim_{x \to 0^+} 3\right)\left(\lim_{x \to 0^+} x\right)^2 + \lim_{x \to 0^+} 2}{\lim_{x \to 0^+} x} &= \quad \frac{2}{0}.
\end{aligned}
$$

The conclusion is arithmetically undefined, that is, we cannot divide by zero except **in**side a limit. This hints at the following solution which does work.

$$
\begin{aligned}
\lim_{x \to 0^+} \frac{3x^2 + 2}{x} &= \\
\lim_{x \to 0^+} \frac{1}{x} \cdot (3x^2 + 2) &= \quad \text{multiplication} \\
\lim_{x \to 0^+} \frac{1}{x} \cdot \lim_{x \to 0^+} (3x^2 + 2) &= \quad \text{addition} \\
\lim_{x \to 0^+} \frac{1}{x} \cdot \left(\lim_{x \to 0^+} 3x^2 + \lim_{x \to 0^+} 2\right) &= \quad \text{multiplication} \\
\lim_{x \to 0^+} \frac{1}{x} \cdot \left(\left[\lim_{x \to 0^+} 3\right]\left[\lim_{x \to 0^+} x\right]^2 + \lim_{x \to 0^+} 2\right) &= \quad \infty
\end{aligned}
$$

The conclusion is the result of known forms including the infinite form $\lim_{x \to 0^+} \dfrac{1}{x} = \infty$.