Correct Handling of Complex Limit Substitutions

$$\lim_{x \to \infty} \frac{-2}{1 - \sqrt{1 - \frac{1}{x}}}$$

If we think ahead on this problem we realize that
$1/x \to 0$ so
$1 - 1/x \to 1$ and
$\sqrt{1 - 1/x} \to 1$ and
$1 - \sqrt{1 - 1/x} \to 0$ so
$1/(1 - \sqrt{1 - 1/x}) \to \infty$
but we don't know if that is positive, negative, or undefined (both). What we are doing mentally are the sequence of substitutions below.

x_2	$=$	$\lim_{x \to \infty} \frac{1}{x}$	$=$	$0.$
x_3	$=$	$\lim_{x_2 \to 0^+} 1 - x_2$	$=$	$1.$
x_4	$=$	$\lim_{x_3 \to 1^-} \sqrt{x_3}$	$=$	$1.$
x_5	$=$	$\lim_{x_4 \to 1^-} 1 - x_4$	$=$	$0.$
x_6	$=$	$\lim_{x_5 \to 0^+} \frac{1}{x_5}$	$=$	$\infty.$
		$\lim_{x_6 \to \infty} -2x_6$	$=$	$-\infty.$

Since x is always positive, this is descending toward 0.
Since x_2 is shrinking, this is growing toward 1.
Since x_3 is growing this is also growing toward 1.
Since x_4 is growing, this shrinking toward 0.

That is the final substitution, so this complex limit is $-\infty$.