1. Calculate $\lim_{x \to 3} 2x$.

   We guess that $\lim_{x \to 3} 2x = 2(3) = 6$. To demonstrate this, we use the epsilon-delta definition.

$$|2x - 6| < \epsilon.$$

$$\begin{array}{rcl} -\epsilon & < 2x - 6 < & \epsilon. \\ -\epsilon + 6 & < 2x < & \epsilon + 6. \\ -\frac{\epsilon}{2} + 3 & < x < & \frac{\epsilon}{2} + 3. \\ -\frac{\epsilon}{2} + 3 - 3 & < x - 3 < & \frac{\epsilon}{2} + 3 - 3. \\ -\frac{\epsilon}{2} & < x - 3 < & \frac{\epsilon}{2}. \end{array}$$

$$|x - 3| < \frac{\epsilon}{2}.$$

   Thus for an arbitrary ϵ we know that $\delta = \frac{\epsilon}{2}$ satisfies the definition.

2. Calculate $\lim_{x \to 3} 3x - 4$.

   We guess that $\lim_{x \to 3} 3x - 4 = 3(3) - 4 = 5$. To demonstrate this, we use the epsilon-delta definition.

$$|3x - 4 - 5| = |3x - 9| < \epsilon.$$

$$\begin{array}{rcl} -\epsilon & < 3x - 9 < & \epsilon. \\ -\epsilon + 9 & < 3x < & \epsilon + 9. \\ -\frac{\epsilon}{3} + 3 & < x < & \frac{\epsilon}{3} + 3. \\ -\frac{\epsilon}{3} + 3 - 3 & < x - 3 < & \frac{\epsilon}{3} + 3 - 3. \\ -\frac{\epsilon}{3} & < x - 3 < & \frac{\epsilon}{3}. \end{array}$$

$$|x - 3| < \frac{\epsilon}{3}.$$

   Thus for an arbitrary ϵ we know that $\delta = \frac{\epsilon}{3}$ satisfies the definition.

3. Calculate $\lim_{x \to 3} (2x) + (3x - 4)$.

   We guess that $\lim_{x \to 3} (2x) + (3x - 4) = 2(3) + 3(3) - 4 = 11$. To demonstrate this, we use the epsilon-delta definition. We modify our technique, however, to use the work done above.

$$\begin{array}{rl} |(2x) + (3x - 4) - 11| & = \\ |(2x) + (3x - 4) - (6 + 5)| & = \\ |(2x - 6) + (3x - 4 - 5)| & = \\ |(2x - 6) + (3x - 9)| & \leq \\ |2x - 6| + |3x - 9|. & \end{array}$$

   The last step is using the triangle inequality.

   We want to find a δ so that $|(2x - 6) + (3x - 4 - 5)| < \epsilon$ when $|x - 3| < \delta$. We know from the first two limit problems that $|2x - 6| < \frac{\epsilon}{2}$ when $\delta_f = \frac{\epsilon}{4}$. Also, $|3x - 9| < \frac{\epsilon}{2}$ when $\delta_g = \frac{\epsilon}{6}$. Because $\frac{\epsilon}{6} < \frac{\epsilon}{4}$, it is also true that $|2x - 6| < \frac{\epsilon}{2}$ when $\delta_f = \frac{\epsilon}{6}$.

   Thus

$$\begin{array}{rcl} |[(2x) + (3x - 4)] - 11| & = & \\ |(2x - 6) + (3x - 4 - 5)| & \leq & \\ |2x - 6| + |3x - 9| & < & \frac{\epsilon}{2} + \frac{\epsilon}{2} \\ & = & \epsilon \end{array}$$

   when $\delta = \frac{\epsilon}{6}$.