

Math 231 Introduction to Discrete Mathematics

Exam 3 Key

Instructions

1. Do NOT write your answers on these sheets. Nothing written on the test papers will be graded.
2. Please begin each section of questions on a new sheet of paper.
3. Please do not write answers side by side.
4. Please do not staple your test papers together.
5. Limited credit will be given for incomplete or incorrect justification.

Questions

1. Graph Properties (2 each)

Determine the following parameters and properties of the graph G in Figure 1.

- (a) Min and max degree

$$\Delta(G) = \delta(G) = 4$$

- (b) Max clique (give size and list vertices)

$$\Omega(G) = 3. \quad \{A, E, L\}$$

- (c) Max vertex independent set (give size and list vertices)

$$|\{A, F, C, J\}| = 4.$$

- (d) Max edge independent set (give size and list edges)

$$|\{\{A, B\}, \{E, F\}, \{L, K\}, \{J, I\}, \{G, H\}, \{C, D\}\}| = 6.$$

- (e) Max n for which it is n -connected (vertex; give n and list vertices that disconnect the graph)

$$4 \quad \{E, L, B, D\}$$

- (f) Give an Eulerian trail if it exists (list vertices in order) or say it does not exist.

A-B-C-D-A-E-F-B-G-F-J-D-K-J-I-C-H-I-E-L-H-G-K-L-A

- (g) Specify a spanning tree (highlight edges on graph).

See Figure 2.

2. Isomorphism (4 each)

Determine which if any of the graphs in Figure 3 are isomorphic. Give the isomorphism if it exists or explain why a pair are not.

Graphs H_1 and H_2 are not isomorphic because H_1 has no triangles and H_2 has $E - F - D$ as one triangle.

Graphs H_1 and H_3 are isomorphic.

A	B	C	D	E	F	G	H
A	F	B	G	C	H	D	E

3. Think a Moment (4 each)

- (a) If a graph G has exactly one spanning tree, what else can you say about the graph?

The graph is a tree. It is connected, because a spanning tree is connected and contains all vertices. It does not have a cycle, because if there were one edge would be excluded, but there would be at least two such choices (spanning trees).

- (b) $K_{2,3}$ is n -connected (vertex) for what n ?

$n = 2$ because removing the two vertices in the same set will leave three disconnected vertices.

- (c) What is the minimum number of edges that can be removed to disconnect $K_{4,7}$?

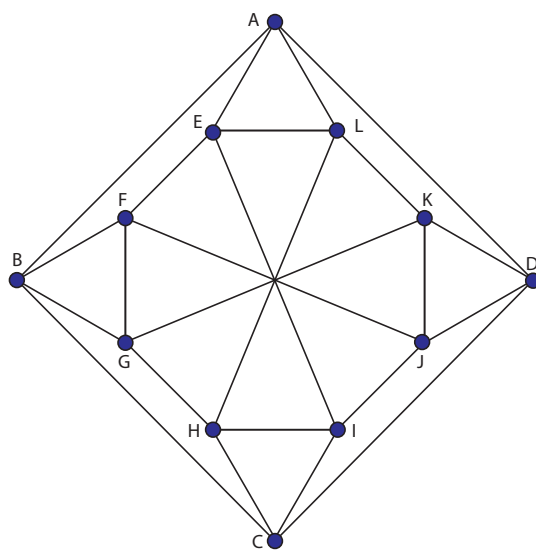
To disconnect a single vertex requires removing the four edges from a vertex of the 7 set.

- (d) For what n, m does $K_{m,n}$ have a Hamiltonian path?

Only when $m = n$ or $m = n + 1$. If one set has two or more than the other then at least one of those cannot be reached, because there is no edge back to an unused vertex.

- (e) Explain why the complement of $K_{n,m}$ is or is not connected.

Because all edges from one set to the other exist, when the complement is constructed there will be no edges from one set to the other. Thus the complement of $K_{n,m}$ is disconnected.

Figure 1: Graph G

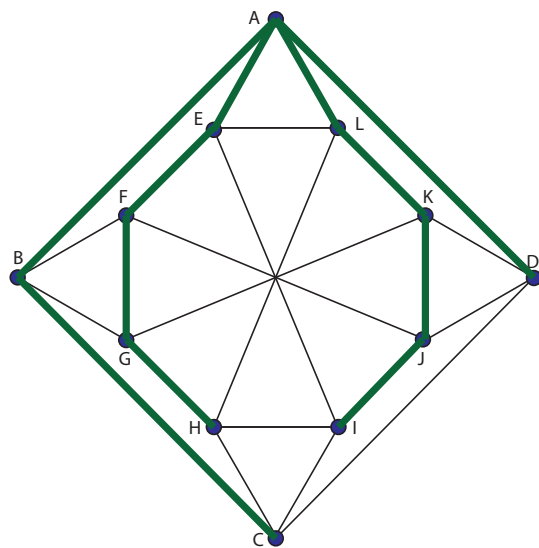
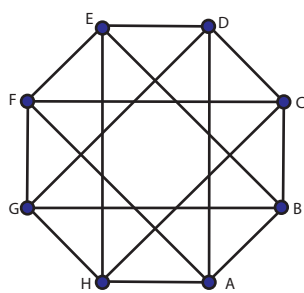
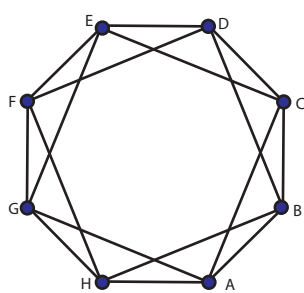


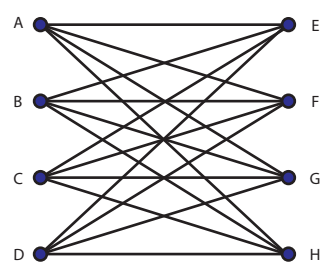
Figure 2: Spanning Tree



Graph H_1



Graph H_2



Graph H_3

Figure 3: Isomorphic