Inverse Functions

We now know that a relation is a function when each input produces exactly one output. Now we want to think about this issue in reverse. Consider $y = 3x + 7$. For $y = 2$

\[
\begin{align*}
y &= 3x + 7. \\
2 &= 3x + 7. \\
-5 &= 3x. \\
-\frac{5}{3} &= x.
\end{align*}
\]

There is only one input for this output. For $y = 7$

\[
\begin{align*}
7 &= 3x + 7. \\
0 &= 3x. \\
0 &= x.
\end{align*}
\]

There is still only one input for this output. We can see that each output will produce exactly one input. This function has an inverse function.

Consider $3y = |x + 1|$. For $y = 2$

\[
\begin{align*}
3y &= |x + 1|. \\
6 &= |x + 1|. \\
6 &= x + 1. \\
5 &= x. \\
6 &= -(x + 1). \\
6 &= -x - 1. \\
7 &= -x. \\
-7 &= x.
\end{align*}
\]

For this one output there are two inputs. Thus this function does not have a function inverse.
For the first example we can find the function inverse. Because the inverse reverses the role of input and output, we can simply solve for the input.

\[
y = 3x + 7.
\]

\[
y - 7 = 3x.
\]

\[
y - 7 = 3x.
\]

\[
\frac{y-7}{3} = \frac{3x}{3}.
\]

\[
\frac{y-7}{3} = x.
\]

This is the inverse function which we also can write as \( f^{-1}(x) = \frac{x-7}{3} \).